Abstract

With the rise of ecommerce on the internet and, especially, the increase in volume of business procurement carried out electronically, double auctions have received much attention in the economics and, increasingly, computer science literature. However, there is no such thing as “the double auction”. Instead there is a profusion of terminology and a wide variety of different market institutions that are all, confusingly, described as “double auctions”. This paper aims to bring some order to this profusion. First, the paper documents the terminology and to identify the variety of market institutions that are grouped under the “double auction” heading. Second, the paper describes some of the theoretical and experimental work that has been carried out on double auctions. Finally, the paper sketches some of our recent work which aims to synthesize a common set of experiments from those detailed in the literature.

1 Introduction

From some of the descriptions of the double auction institution that one often finds in the literature, it is easy to get the impression that “the double auction”
is a single concept like the English or Dutch auction\(^1\). However, on looking more closely into the matter (for example when trying to implement such a market), it quickly becomes clear that there is a wide range of different variants of the double auction. The first aim of this note is to lay out some of these variations, and, in particular, the differences between them. The second aim of this note is to describe some of the theoretical and experimental work that has been carried out on double auctions. Finally, the third aim of this note is to sketch some of our recent work which aims to synthesize a common set of experiments from those detailed in the literature.

1.1 Terminology

We start by considering the description of the double auction market given by Friedman \cite{7}, which kicks off the influential volume \cite{8} describing work from the Santa Fe Institute conferences on double auctions which were held in 1990 and 1991.

We deal with interactions between agents, traders, and commodities that they own. We will usually deal with two commodities, the \textit{good} and \textit{money}. Sometimes this will be extended with additional kinds of good. All goods are taken to be indivisible, while money is divisible. The interactions between the agents lead to exchange, a process whereby the traders freely alter the allocation of commodities, without changing the total quantity of the commodities. Thus traders may choose to swap a good for some quantity of money, and the amount they choose is related to the value the trader places on the good. This value is known as the trader’s \textit{private value} or \textit{limit price}. The latter terminology comes from the fact that if a buyer pays more than its private value, or a seller accepts less, then it considers it is trading at a loss.

A \textit{market institution} defines how this exchange takes place. It does this by laying down rules about what the traders can do (which comes down to specifying what messages they can exchange) and rules for how the final allocation of commodities given the actions of the traders. When the allocation of commodities changes, a process known as the market \textit{clearing}, the difference between the final and initial allocations is the \textit{net trade}, and this has a component for each trader in the market which shows the change in the amount of money and good that they hold. The minimal net trade is when two traders have non-zero components, and any trader that has a non-zero component has an associated \textit{transaction price} which we will also sometimes call the \textit{trade price}—the absolute value of the money component divided by the good component\(^2\). Traders with positive good components are \textit{buyers}, those with negative good components are \textit{sellers}. Typically we will deal with so-called one-way traders, meaning that traders are either buyers or sellers but not both.

So far we have not said much about what form the market institution takes. Friedman \cite{7} defines an \textit{auction} as a market institution in which messages from

\(^1\)Though there are variants of both the English and Dutch auctions, the range of this variation is small.

\(^2\)Neither component is allowed to be zero if the other is zero.
traders include some price information—this information may be an offer to buy at a given price, in the case of a bid, or an offer to sell at a given price, in the case of an ask—and which gives priority to higher bids and lower asks. We can allow only buyers or only sellers to make offers, in which case the market is one-sided, or we can allow both, in which case it is two-sided. A double is a two-sided auction, and from here on we will only deal with double auctions (though many of the distinctions we identify could apply to other kinds of market as well)³.

1.2 Structure

Now, given this set of definitions, we are ready to proceed with the cataloging of the double auction literature. The structure of this paper is as follows. Section 2 identifies the varieties of market that one can find in the literature, and Section 4 then briefly surveys experimental work on the double auction, leaning heavily towards work that uses artificial agents rather than human traders. Finally, Section 5 describes some of the work we have been undertaking which aims to replicate and relate work described in previous sections. Finally Section 6 concludes.

2 Types of double auction

The aim of this section is to suggest a framework into which all existing work on double auctions can be fitted, and to then identify into which part of the framework that existing work fits. This suggested framework is not really new, but a synthesis of suggested terminology and partial classifications that are scattered across the literature.

2.1 Attributes of double auctions

A double auction can be one-shot or it can be repeated. If it is repeated (and as we will see below, most experimental work has looked at repeated auctions), then the auction has several, as opposed to one, trading periods. The reason that it is worth distinguishing repeated auctions as opposed to one-shot auctions is that at the start of each trading period agents receive a new allocation of commodities (thus modelling “conditions of normal supply and demand” [32]) but have the scope to remember the results of trading in the previous period. Thus, after several periods, we can expect the market to reach some kind of steady state. Of course, running a number of periods with the same supply and demand curve is an experimental nicety — in real auctions supply and demand curves are no static, and some work has considered such cases. For example [13] and [21] look at changes in supply and demand in the context of the New York Stock Exchange, and [10] have examined the performance of artificial trading agents under such conditions.

³Friedman [7] distinguishes quasi-auctions as well as other kinds of market—a quasi-auction has most of the features of an auction but not all.
Figure 1: Illustrative supply and demand curves for a double auction.

In a periodic or discrete time double auction, clearing happens some fixed period of time after the auction starts. This clearing might happen several times in each trading period, or may happen just once at the end of the trading period. The classic example of a periodic double auction is the call market or clearing house used to determine the opening prices on the New York Stock Exchange. Traders submit bids and asks until the end of the trading period and these are used to determine the supply and demand curves for commodities. Example supply and demand curves, taken from [1], are shown in Figure 1. These are based on having traders A, B, C, D and E all looking to supply 10 goods with limit prices of $15, $20, $25, $30, and $35 per good respectively, and F, G, H, I, and J looking to buy with limit prices of $15, $20, $25, $30, and $35 respectively. If the trade price is set below $15, no seller will be prepared to trade, while if the price is between $15 and $20 then only A will trade, and so on. Similarly, at a price above $35 no buyer will trade, while at prices between $30 and $35 only J will trade, and so on.

Once these curves are established, they are used to set a price for trading, the price at which supply equals demand in this case. The price at which supply equals demand is known as the equilibrium price, and in the case of the market in Figure 1 is a price of $25. In this kind of auction, it is an institution external to the traders—the clearing house that gives the market one of its names—that is the entity that decides the trade price. In other kinds of auction the traders themselves set the price, and we will distinguish the two cases as those institutional price-setting and non-institutional price-setting respectively. Similarly either the traders themselves or the institution can decide when allocations will change. The traders can do this by indicating acceptance of

\[\text{Figure 1: Illustrative supply and demand curves for a double auction.}\]

4Though “call market” and “clearing house” seem to be the most common names for a periodic market, [18] also class such entities “batch markets”.

5Though as [20] explains, as as we will discuss below, the New York Stock Exchange doesn’t work quite like this.
a bid of ask, and the institution can do this by encoding it in a rule, for example that traders whose bid and ask values cross (in other words where a buyer is willing to pay more than a seller is asking for) must trade immediately. We distinguish these cases as institutional trade-determination and non-institutional trade-determination respectively. Both of these are aspects of clearing.

Whatever form of clearing is used, the effect of the periodic form of the double auction is that all traders establish their final allocations at the same time. (The final allocation may be the same as the initial allocation of course). The continuous or open-outcry double auction, in contrast, does not have a specified time for clearing. Instead, in non-institutional trade-determination, buyers and sellers can choose to accept a bid or ask, and then update their allocation, at any point in time. In institutional trade-determination, the institution effectively has to try to clear the auction (which means find a bid and ask that cross) any time a new bid or ask is made by a trader. As Friedman and Rust [9] note, the New York Stock Exchange traded as a call market until 1860 and its move to continuous trading was driven by growth in trading volume, suggesting that it is superior throughput that is the reason for selecting the continuous rather than the periodic form of the double auction. However, there can be a cost for this increased throughput—as Gode and Sunder [12] show for their zero-intelligence agents (see below) the expected efficiency of a call market is higher than that of a continuous double auction, and is is easy to verify that this will be the case for other trader behaviors as well.

The rules of the institution thus define the way that the market clears, and the division between markets using different kinds of clearing is the most obvious distinction to be made between double auction variants. However, there are other important distinctions. One of these is what conditions, if any, that the market puts on successive bids and asks. Gode and Sunder [11] give a typical rule when they say that:

\[ \ldots \text{any buyer can enter a bid by stating his or her identity, unit price and quantity. The same buyer or other buyers can subsequently raise the bid.} \]

Here unit price is the price that a buyer is prepared to pay for a single good, and this is constrained to increase from bid to bid until a transaction occurs. Such a restriction is called a bid/ask improvement rule in [12]. Cliff [1] notes that since this rule is enforced by the New York Stock Exchange, (NYSE) it is often referred to as the NYSE rule.

Obviously, in order to be able to ensure that they make a higher bid (or lower ask which is the natural complement of a higher bid), every agent needs to know what other agents bid and ask. However, this kind of knowledge is not always available. In a classic open-outcry market (and here the classic market is the commodity trading pit at the old Chicago Board of Trade [9]), every other pit trader is able to hear every bid and ask made (or at least has the potential to do so). However, less privileged traders [7] do not have access to current trades. Something similar is the case at the New York Stock Exchange, where specialists [14] have complete access to all bids and asks for a type of good (and there is one
specialist for each type), while floor traders are allowed to check the unmatched bids and asks, so some will periodically have this information while others will only know the value of the highest unmatched bid and lowest unmatched ask ([37] states that this is the most common information provided by a continuous double auction). This latter kind of information is known as the bid-ask quote. It is also possible to imagine traders just having information about the price at which the last trade was conducted (which bounds the winning bid and ask rather than giving it precisely) or having no information at all (so any offer has to be based on the trader’s private value alone). Taking all of these together we can characterise an auction by the kind (or kinds) of information provided to the traders.

In a continuous double auction it makes sense for traders to get some information about prices, and indeed this kind of feedback is seen as “a major feature” [32] since it is the only way a trader can identify how others value the good and therefore decide how to bid (in other words bids and asks are taken as indications of private values) and as Rust et al. [29] point out, this allows auction participants to learn about one another’s preferences. Although some call-markets do not issue price quotes—Friedman and Rust [9] point out that this effectively reduces the call market to a kind of sealed-bid auction—it makes sense for them to offer such information for the same reason. The importance of being able to learn is hard to overemphasize. The double auction, whether continuous or periodic (provided there is more than one period):

\[
\text{\ldots appears to be too complex a game to yield a clear game-theoretic solution} \text{ [12]} ^6
\]

Since it is so hard to predict how other traders will behave in this kind of market, it is not possible to figure out the best way to behave before the auction starts —the only way to determine the best way to behave in a double auction is to learn that behaviour as the auction unfolds.

Another related issue is what happens to unmatched offers. Clearly if a trader changes its bid or ask, then the old one will be discarded, but should anything be done to ones that are (or haven’t yet been) updated at the time the auction clears? Unlike the situation with price quotes, there are just two choices. Either the unmatched bids are left open until either they are filled or the auction closes, giving a persistent shout auction—a feature that [33] argue is true of real continuous double auctions—or they can be cleared as in [12] and [1].

The first quote from Gode and Sunder [12], above, also illustrates another distinction that can be made between types of double auction. Some are, implicitly or explicitly, single unit markets which only allow traders to bid or ask for individual items at a time. Others are multi-unit markets which allow traders to bid and ask on several units (and indeed change the number of units between offers).

---

\[^6\text{This sentiment is echoed by [29], and supported by the fact that all attempts to analyse double auctions that we are aware of, for example [12, 31, 37], only try to analyse call markets and do so under very restrictive assumptions.}\]
Another important distinction between different markets is what mechanism is used to establish the price at which a trade occurs (as opposed to when a trade can occur and if it does when it can, as discussed above). These mechanisms, in the main, make most sense where price-making is institutional, though presumably traders could take some joint decision about adopting such mechanisms. In situations such as that in Figure 1, there is a single price at which trading will be acceptable, $25, and so this will be the trading price. However, in the market described by Figure 2, there is a price tunnel between $20 and $25—any price in this range will be acceptable. Price setting mechanisms are required because in general supply and demand curves overlap as in Figure 2.

In call markets, it is typically the case that several buyers and sellers have bids and asks that match when the market clears. Since they are all trading identical goods (at least in all the markets and studies we are considering here this is the case), there are two obvious ways we can decide the trading price for each matched pair—either we can choose one price and apply it to all trades, or we can choose prices for each trade individually. The first is generally termed a uniform price [37], the latter a discriminatory price. One can easily see arguments for both choices. Uniform price auctions ensure that no trader is discriminated against because of a wayward bid (caused by a misreading of the market for example)\(^7\). Discriminatory price auctions ensure that the price a trader pays is determined by its bid and that of its trade partner, and supposedly this reflects their amount that they value the good being traded.

Continuous double auctions are, almost by definition, discriminatory-price, although it is possible to imagine some kind of institutional price setting that matches traders continuously, but then sets the trade price uniformly at the end of the trading “day”, perhaps to the equilibrium price that the market reaches\(^8\).

\(^7\)Though its bid will typically help to set the trading price, through a mechanism like picking the \(m\)th price.

\(^8\)We have recently [23] been looking at a form of auction which, though discriminatory
Whether an auction is uniform or discriminatory priced, it needs to have a mechanism for choosing the price, and there is no standard mechanism for doing this. Instead a number of different mechanisms have been proposed and studied in the literature. [37] studies the application of the mth price mechanism to a uniform-price call market—here the trading price is that of the mth highest bid at clearing, a price that will be less than or equal to the highest matched bid (obviously) and thus greater than or equal to the lowest matched ask. [31] also studies the call market, but considers the k-double auction mechanism rather than the mth price mechanism to set the trade-price between the highest bid and lowest ask. For the k-double auction mechanism we choose a value k between 0 and 1, and then set the price to be:

\[ kb + (1 - k)a \]

where \([a, b]\) is the interval in which a price can be selected. One could either apply this mechanism in a uniform-price auction (where the interval is taken across all matching bids and asks) or in a discriminatory auction, and it is also possible to use it in a continuous double auction. [37] call the parameter \(\kappa\) rather than \(k\) when it is used in a discriminatory rather than a uniform-price rule.

A \(k\) or \(\kappa\) double auction treats all trades of a given type alike — the parameter sets the fraction of profit that goes to buyer and seller, and all buyers and sellers get that fraction. [11] describe another mechanism. In this latter approach, the price of a trade is the price of the bid or ask that was made first, and so it “on the table” when the match occurs. Clearly this will allocate all of the profit in the transaction to the trader who made the deal possible, and this trader can be either buyer or seller.

Now, so far we have assumed that all trading takes place between peers—between agents that either buy or sell—and that the auctioneer organises the market but does not trade. However, there are markets in which the role of auctioneer is taken by a market maker who also makes trades. [18] distinguishes between quote-driven and order-driven markets. The latter are the kinds of auction that we have discussed up to now, markets in which traders submit orders and these are either matched with existing orders, or wait on an order book until a matching order is received by the auctioneer. Quote-driven systems, like those that operate on NASDAQ and the London Stock Exchange (LSE), are continuous dealer markets in which trades who wish to execute an order can trade directly with the market maker—it is part of the market maker’s job to provide liquidity in this way, always buying or selling good on demand. While the continuous dealer markets on the LSE, NASDAQ and the specialist part of the NYSE have only one market maker per type of stock (the specialist for that stock), other markets, for example the Chicago Board of Options Exchange (CBOE), operate as multiple dealer markets [17] where several market makers operate and traders choose which one to deal with.
So, to summarise, we can distinguish between double auctions by answering the following questions:

- Is the market a quote-driven or an order-driven (auction) market?
- If the market is quote-driven, is it a specialist market or a multiple dealer market?
- Is the market one-shot or repeated?
- Is it periodic or continuous?
- Are prices set by an institution or by the traders?
- Is the decision to trade taken by the traders or by the institution?
- What conditions, if any, are placed on successive bids and asks (is there a bid/ask improvement rule)?
- What information about other traders offers does each trader have?
- What happens to unmatched bids and asks when a match occurs (are they deleted, or do they persist)?
- How many goods are bid (or asked) for at a given instant by one of the traders (one or many)?
- How is the trade price is determined?

2.2 Categorising double auctions

In this section we use the attributes identified above to categorise some of the double auctions discussed in the literature.

We start with Smith’s classic study of competitive market behaviour [32]. Here the traders were all human, and the auction was a true continuous double auction (by which we mean the kind of market that one would find on the trading floor of the NYSE). Traders are given specific limit prices (all of which are specified in the paper) and can make a bid or ask at any time. Offers that cross are cleared at a price that is midway between the two offers (implementing a $\kappa = 0.5$ pricing rule). Offers are persistent (though presumably traders forget old offers), and are run over several periods (the number varies from experiment to experiment, but is typically around 4 or 5).

The auction used in the Santa Fe Double Auction tournament [28, 29] is repeated (it is broken into a number of “rounds”), and is a hybrid of a periodic and continuous auction. The auction is structured as a series of “steps”, where one step allows bids and asks to be posted and the subsequent step clears the market by matching the highest bid and lowest ask. The trading period is a number of these steps, and a number of these periods make a “round”, of which there may be many. The decision to trade is made by individuals, though
only the matched individuals are allowed to trade⁹, and it is not clear how the transaction price is set, though a bid/ask improvement rule does seem to be enforced. In addition it seems (according to [33]) that unmatched bids are cleared when transactions occur, that all traders have access to all the offers made by all the other traders, and that only one good is bid for at a time.

Gode and Sunder’s work [11] on zero-intelligence agents concerns repeated auctions (and each auction but one in their study were run for six trading periods), and the auction was continuous. Trade determination is institutional, in the sense that a binding transaction occurs as soon as a bid and ask cross, and price determination also seems to be institutional—the price is determined by whichever of the bid or ask is already “on the table” when matching occurs. In addition, unmatched offers are cleared when a transaction occurs, and only one good is bid for at a time. All of these factors are clear from the description of the experiments.

The remaining factors are less clear from the paper. It seems that though Gode and Sunder consider a bid/ask improvement rule to be an essential part of a double auction, there seems to be no mechanism for it in the automated traders they discuss (indeed since these are described as having “no memory” it would seem impossible for such a mechanism to be implemented), and it is not clear whether such a rule is applied to the human traders in their study either. Furthermore, it is not clear what information traders have. From the description of the experiment with human traders, it seems inevitable that the human traders knew all the offers that had been made. There is nothing to say whether the automated traders had this information, but if they did, then the mechanism they used for choosing bids and asks does not make use of it.

In Cliff’s work on zero-intelligence plus (ZIP) agents [1, 4, 2, 3]¹⁰ the auction is repeated and continuous, and there are no persistent shouts—all bids and asks are cleared when a trade takes place. Furthermore, as described in [26], the way that the auction is simulated is quite specific. At each simulation “round”, one agent is randomly selected to make an offer. All other agents are made aware of this offer, and can respond by accepting it (and trading thus takes place at the price suggested by the first agent just as in [11]. If several agents respond, one is chosen at random to complete the trade. Thus the market is synchronised, with every trader guaranteed to be given the chance to make an offer each round.

[26] builds directly on Cliff’s work, and so uses much the same structure. Time is again divided into “rounds”, with all agents having to make an opening offer at the first round, and any agent being able to update its offer at a later round (unless of course it has traded). Offers persist unless updated. Offers that cross are cleared just as in Smith’s work, and Preist and van Tol argue

---

⁹The same rules as adopted by the AURORA system developed by the Chicago Board of Trade.

¹⁰These four papers overlap to a large degree. Without knowing the history of their writing, it seems that [1] was written first as a complete summary of the original work (including a listing of the code used to run the simulations), [4] was a slimmed down version without the background on market-based control and background economics, and then [2] and [3] were then extracted out as conference papers (the former introducing ZIP traders and the latter identifying problems with zi-c traders).
both that their market is close to Smith’s and is more flexible than Cliff’s.

What we can see from this cross-section of work is that research has aimed to be representative rather than strictly reproducing markets that really occur—for example the fact that many approaches cancel all unfulfilled orders when a trade is made—but has covered a range of subtly different markets, something that makes it hard to directly compare the results of the work. This problem is exacerbated, as we will see below, because the performance of the traders is determined in different ways as well.

3 The New York Stock Exchange

Have described in the abstract some of the different kinds of double-sided market, it is interesting to consider a concrete example of such a market in the form of the New York Stock Exchange (NYSE) [15]. As hinted at above, the NYSE is rather more complex than anything we have described so far.

Physically the NYSE is split into the upstairs and downstairs markets, where the downstairs market is the main trading floor (we will come back to the upstairs market later). The main trading floor is split into a number of rooms 11, throughout which are spread seventeen trading posts. It is at the trading post for a specific stock that the specialist for that stock works, and it is at the post that the trading crowd for that stock will gather. The trading crowd is made up of floor brokers. Floor brokers are allowed to trade in any stock and to operate at any trading post, but typically concentrate on a few posts and trade only in the stocks traded at those posts.

Trading on the NYSE takes place between 9.30am and 4pm EST Monday to Friday. After opening, trading in each stock takes the form of a continuous auction 12, with each specialist receiving all offers for the stock in which they deal, and maintaining the order book that records such offers. The specialist supervises the trading process, makes matches between buyers and sellers, and is responsible for crowd control amongst the trading crowd. As described above, the specialist is required to trade when necessary, and can also trade when it is advantageous to do so, but as [20] points out, the volume of trade involving specialists is now a minority, less than 20% by 2000. The vast bulk of trades are direct between brokers.

When the exchange is not open for trading, orders continue to arrive electronically, and are stored in the Opening Automated Report Service (OARS), which matches bids and asks. In addition, prior to opening, floor brokers who want their orders to figure in the opening may pass orders to the relevant specialist, and these are also entered into OARS 13. When the market opens, the

---

11The Garage, the Main Room, the Blue Room and the Expanded Blue Room [15].
12Though as [18] puts it, “for some thickly traded stocks, the market has been described as a continuous double auction” which suggests that the market in most stocks is too thin to be regarded as a true CDA.
13As described in [20], brokers may make orders for the open without being physically present, and may choose to participate in a certain percentage of the specialist’s opening trade instead of placing a standard limit order.
The role of the specialists in the NYSE is studied in detail by [14], who both expand on their role in the institution, and examine empirical evidence about their activity in the market (which, for example, suggest that they are typically good short-term traders but not particularly good long term traders). Specialists are required to maintain price continuity—which basically limits successive price changes to one eighth of a dollar—and to act to stabilize the market, buying when the price drops and selling when it rises. These requirements expose the specialist to sudden markets shocks, but this disadvantage is offset by the fact that the specialist has complete knowledge of the order book—the specialist has to allow floor traders to view the order book, but the book is not widely displayed so the specialist has a significant advantage in terms of information [20]—and gets to see incoming electronic orders before these are posted in the order book.

The upstairs market [19] is an informal market that aims to deal with large blocks of shares (where a “large block” is well over 10,000 shares) by negotiating a price between potential buyers and sellers. The idea seems to be that it may be hard for the trading floor to absorb such blocks, though as [15] points out, only around 15% of total trade is arranged through the upstairs market.

There is one final quirk of the NYSE that is worth noting. The exchange itself is a not-for-profit organization, founded originally in 1792, that is owned by those who trade in it, so-called seatholders\textsuperscript{14}. Seats are limited—there were originally 1,060 seats, and this number was expanded to 1,375 in 1932 and 1,366 in 1953—and seatholders are the only people allowed to operate on the floor of the exchange, either as specialists, floor traders or brokers as they wish\textsuperscript{15}. Since 1978 seats may be leased, and this is now relatively common, but seats may also be bought and sold (seats were first traded in 1869), and this trading itself takes place through an ongoing auction market run by the NYSE. This market is theoretically continuous, and the bid and ask quotes are posted on the trading floor, but seats are thinly traded.

4 Experimental work on double auctions

Having categorised some of the auctions that have been studied in the literature, we discuss the experimental work that has been performed using some of those auctions in more detail. This leads on to some analysis of the way in which one can quantify auction performance.

\textsuperscript{14}Up to 1871, members sat in assigned seats during the roll call of stocks.

\textsuperscript{15}There are other roles, as described in [16] but these need to concern us here.
4.1 A brief survey of experimental results

The work that started the experimental study of double auctions is that of Vernon Smith, and the first paper on this work is [32]. This work deals only with human traders (as one might imagine given when it was carried out), and examines how well markets work when they only have:

\[ \ldots \text{a practical number of marketers} \ldots \]

rather than the “indefinitely large” number previously assumed necessary, concluding that even small numbers of traders allow competitive equilibrium to be achieved. This is demonstrated by the propensity of the transaction price to converge to the theoretical equilibrium price (the price at which the supply and demand curves, defined by trader private values, cross). These results held for a number of different supply and demand curves, suggesting a certain degree of robustness to the results.

One interpretation of this work is that the kind of market provided by the double auction is strong enough that it manages to generate efficient outcomes even when assumptions like an “indefinitely large” number of traders are violated. Gode and Sunder [11] followed this up by investigating whether the market could cope with traders who made unintelligent offers. In particular, they looked at two varieties of what they termed zero-intelligence agents. Both of these bid randomly. zi-u (unconstrained) agents pick a bid or ask from anywhere within a given price range (up to 200 monetary units) and zi-c (constrained) agents pick offers constrained by private value. Thus zi-c buyers bid below their private value and zi-c sellers ask above their private value, thus ensuring that both trade at a profit.

The results of this investigation were that while prices did not converge to equilibrium (which one would not expect given the traders are picking prices randomly) they did trade close to the theoretical equilibrium price, and the markets with zi-c agents had high allocative efficiency. This is a measure of how effective the market is at generating profits for the traders. For each trade it is possible to determine the profit for each trader (the difference between the trade-price and their private value) and so to establish the total actual profit for the auction. This can then be used to compute the fraction of the total possible profit that the auction could have extracted—if the buyer with the highest private value trades with the seller with the lowest private value, and the buyer with the second highest private value trades with the seller with the second lowest private value, and so on until the private values no longer cross then the maximum possible profit is extracted. The human markets (and Gode and Sunder ran some experiments with human markets with the same private value distributions for comparison) traded at around 100% efficiency and zi-u traders at between 48.8% and 90%, while zi-c traders achieved between 96 and 99.9% (averaging 98.7% across the multiple periods and the different private value distributions)\(^1\).

\(^1\)These results are replicated in Table 1 below.
This would seem to suggest that traders don’t have to have any intelligence (aside from avoiding trading at a loss; the zi-U traders can achieve large negative efficiency if the trades are such that both buyer and seller trade at a loss) at all for the market to be efficient. However, this result has subsequently been challenged. Cliff [1, 4, 3] suggests that zi-c agents trading close to the theoretical equilibrium has more to do with the sets of private values chosen than the power of the market. For a classic upward-sloping supply curve (more units are sold as the price increases) and a downward-sloping demand curve (less units are bought as the price decreases) of Figure 1, Cliff argues that the mean of the probability distribution from which the zi-c agents pick prices is around the theoretical equilibrium price. Thus it is no surprise that for such markets, which are exactly the kind of markets studied in [11] (and most of the markets studied in [32]), zi-c agents trade near the equilibrium price\textsuperscript{17}

Cliff then tested the hypothesis that it is the private values (which determine the shape of the supply and demand curves) that have the biggest effect on the apparent equilibrium price by running simulations of auctions with different supply and demand curves. In addition to the kind of markets studied by Gode and Sunder, Cliff used markets with:

- Downward sloping demand curve and flat supply curve.
- Flat supply and demand curve with excess demand (fewer sellers than buyers and all sellers have a private value below that of the buyers).
- Flat supply and demand curves with excess supply fewer sellers than buyers and all sellers have a private value below that of the buyers).

The results confirm Cliff’s hypothesis—zi-c agents trade a significant distance away from the theoretical equilibrium in these new kinds of market. Note that Cliff reports no results about the efficiency of the zi-c agents, and this need not be low just because trading takes place away from the theoretical equilibrium. To establish the allocative efficiency, we sum all the profit made by all the agents, and divide that by the total possible profit. Thus a buyer $B$ and a seller $S$ who trade at a price $P$ when they have private values $V_B$ and $V_S$, contribute:

$$(V_B - P) + (P - V_S)$$

to the numerator of this computation, and this, of course, reduces to:

$$V_B - V_S$$

so the efficiency depends only on the private values of the traders, not on the price at which they trade (providing the latter is between their private values).

\textsuperscript{17}This effect is exacerbated by the fact that the markets used in [11] are such that there are very few extra-marginal traders—that is traders whose private values are such that they would not trade at equilibrium. In a “blind” market like that populated by zi-c traders, every extra-marginal trader is a chance to lower efficiency and create trades that diverge from the equilibrium price.
if the last bid or ask was accepted at price \( P \) then
\[ \text{if } V_B \geq P \text{ then increase profit margin} \]
\[ \text{if the last offer was an ask} \]
\[ \text{then} \]
\[ \text{if } V_B \leq P \text{ then decrease profit margin} \]
else
\[ \text{if the last offer was a bid} \]
\[ \text{then} \]
\[ \text{if } V \leq P \text{ then decrease profit margin} \]

Figure 3: A part of the ZIP algorithm.

In other words, high allocative efficiency depends only upon matching buyers that have high private values with sellers that have low private values.

In addition to his critique of the Gode and Sunder’s work, Cliff [1, 4, 2] provides an alternative approach, the zero-intelligence-plus (ZIP) trader. This trader listens to the bids and asks being made, and depending on how these relate to its private value, raises or lowers its bid (or ask). For example, a buyer with private value \( V_B \) operates as in Figure 3 where every buyer is bidding above its private value by its profit margin, and every seller is bidding below its private value by its profit margin. The increases and decreases are carried out using a variation of the Widrow Hoff rule [36]. For full details see [1, 4].

Preist and van Tol, colleagues of Cliff’s at HP labs, extended his work in [26]. They started with the observation that one of the assumptions underlying Cliff’s work, that all outstanding bids and asks are cleared after every trade, is somewhat unrealistic for many markets. As a result, they investigated persistent shout double auctions, and adapted the ZIP algorithm to deal with this case. Their new approach, which we will refer to as PVT, was tested against ZIP traders in one of the markets that Cliff introduced in [1] (the symmetric market Cliff calls “A”). Given that Cliff’s interest was with the degree to which traders approach the theoretical equilibrium price, it is natural that [26] should look at this also, and they use the \( \alpha \) measure to show that PVT agents provide a more stable solution (meaning that they have a lower \( \alpha \) measure and they further claim that PVT traders approach equilibrium faster.

All the work considered so far looked at auctions in which only one kind of agent took part (counting human traders as one kind of agent, as studies typically do, despite the fact that they doubtless use a range of strategies). It is also interesting, and arguably more realistic, to investigate auctions in which traders use a variety of bidding strategies.

An interesting example of this line of work is [5], which pits human traders against trading agents. Prior to this work, it had been established that human traders quickly learnt to trade near equilibrium, and could outperform zi-U and
zi-c traders. However, nobody had compared human performance with that of the “more than zero” intelligence traders that had been developed. [5] pitted 6 humans against 6 agents — both types of tarder were both buyer and seller in all their experiments — where the agents used modified versions of GD (MGD) and ZIP (MZP), the modification being necessary to deal with an order book that was not cleared at every trade. MGD is described in some detail, MZIP is said to be similar to PVT. Results, albeit only on six experiments, show that agents tend to be more efficient traders (agents managed to trade at over 100% efficiency, suggesting that they steal profit form the human traders, while the humans traded between 65% snd 97% efficiency).

Comparing a range of automated trading strategies was the aim of the Santa Fe Double Auction Tournament [28, 29]. In order to encourage a variety of entries, the organisers offered cash prizes and attracted 30 submissions. These entries were then played against one another, with one copy of each entry pitted against a single copy of every other entry. In this respect the tournament was the complete opposite of the homogeneous set of traders in both Cliff’s and Gode and Sunder’s experiments. The aim was also different—here the focus was on finding the strategy which gains the greatest profit over the course of many runs of the market (and thus from different private values).

The two top strategies in the tournament were both snipers\textsuperscript{18}, that is strategies which wait until a deal seems settled and then run in and steal it. The top scoring program, named Kaplan in honour of its creator, is also very simple. It does not bid until the gap between the latest bid and ask is less than 10%, and then bids equal to that last ask as long as it is profitable to do so. Such a simple strategy is surprisingly effective, and, as discussed in [28], even manages to make good profits when bidding against large number of copies of itself. This latter fact was established through an evolutionary tournament in which successful strategies were allowed to “breed” and unsuccessful ones were eliminated, though this tournament also revealed that Kaplan traders are not stable—eventually the population of Kaplan traders goes into decline.

Work in a similar vein has recently been carried out by Tesauro, Das and colleagues. In the search for traders which are capable of making high profits, Tesauro and Das [33, 34] exhaustively tested five strategies—zi-c, Kaplan, ZIP, GD, an algorithm proposed by Gjerstad and Dickhaut [10], and MGD, a modified version of GD. GD is a relatively complex trading strategy that estimates how likely bids and asks are at a particular price and adjusts its own offers accordingly. The results show that homogeneous sets of traders of all strategies gain high efficiency (the highest is 0.997 for ZIP and MGD and the lowest is 0.983 for Zi-C), that both ZIP, MGD converge quickly to an equilibrium (results for GD are not given), and that these latter two also come very close to the theoretical equilibrium price on average.

These results, of course, only measure the working of the market as a whole. To find the effectiveness of strategies for individual traders, Tesauro and Das carried...

\textsuperscript{18}A term not in use at the time the experiments were carried out, but familiar to anyone who uses eBay.
ried out further experiments. First they tested whether traders in an otherwise homogeneous population gained or lost by changing strategy. This showed that it was never profitable to change to using zi-c from any of the other strategies, and that it is always better to switch from zi-c. Similarly traders can always benefit by switching to Kaplan, but this benefit is small against GD and MGD traders. A single ZIP traders always do well, except against MGD, and MGD always does well, except against ZIP (and no results are given against GD).

With more balanced groups of types of agent, a number of direct comparisons were made. Over 100 runs, the number of “wins” (where that group gained more profit than the other group) was counted. This showed that zi-c could defeat Kaplan, GD could defeat zi-c and Kaplan, ZIP could defeat GD, zi-c and Kaplan, and MGD could defeat all the others (and by a significant margin). This leads Tesauro and Das to the conclusion that MGD dominates the other trading strategies.

A more sophisticated analysis is the heuristic strategy analysis of [35], work that is a natural extension of that in [33, 34]19. The main idea of [35] is to apply evolutionary game theory [30] to the kinds of results that previous authors had obtained for trading agents. Overall the argument for the evolutionary approach is as follows. Consider a set of traders using different trading strategies, some of these will do better than others, and which ones do well will depend upon the exact mix of traders—this is seen both in [33, 34] and the “breeding” work in [28]. Now, imagine a particular trader, using a specific strategy, after some number of auctions can identify how well traders using different strategies are performing in the current mix (taking profit per trader using a given strategy as a measure of the performance of that strategy). A rational trader will choose to adopt a different strategy if that strategy is performing better, and if there are several such strategies, will pick the one that is performing best. This will change the mix, and after some more auctions the trader may again change its strategy. If we establish the relative performance of different approaches for all the possible mixes (for some finite number of agents) we can determine whether there are stable equilibrium points that the set of traders will converge on, and what the mix is at such equilibria.

Of course, there is a huge number of possible strategies, and hence possible mixes, that we need to consider. The second key idea in [35] is that rather than choose an arbitrary strategy, sensible traders will stick with strategies like GD which are known to perform well. They choose to concentrate on Kaplan, ZIP and MGD, and looked at versions of the CDA with 6, 12, 14, 16, 18 and 20 agents. Results show that in the 6, 12 and 14 agent CDA the Nash equilibrium is a mixed strategy in which MGD has zero probability, while the 20 agent game has three Nash mixed-strategy equilibria, in two of which MGD is the most likely strategy to be played. Furthermore, altering the payoffs slightly (to simulate an improved strategy) suggests that a modest improvement in the payoff to MGD would result in it becoming nearly dominant.

19Which is not to say that it is unoriginal—the work was strikingly original, and only seems like an obvious extension in retrospect.
We have extended this approach in two ways. First [25] we have used the heuristic strategy approach to establish a measure of how good an auction is. The analysis gives us the equilibria, and knowing what the allocative efficiency of each trading strategy is in the equilibrium, we can compute the efficiency in equilibrium. Where there are several equilibria, we can use the size of the “basin of attraction” to establish the probability of the equilibrium, and so compute the expected efficiency of the auction mechanism. Second [24], we have used the approach to guide the automatic creation of new trading strategies. Starting with an evolutionary analysis of MGD, truth-telling (TT) (honestly reporting private values) and the Roth-Erev (RE) adaptive strategy which mimicks human performance [6, 22, 27], we show that while MG is initially nearly dominant, we can evolve strategies that are competitive for certain mixes. Figure 4 shows the result of the analysis before and after a new strategy is evolved.

4.2 How to measure performance of auctions and traders

The reason behind much of this experimental work has been to try and quantify the performance of either auctions or traders. we have already discussed allocative efficiency which is one measure of the quality of an auction—from the perspective of the designer of an auction, or the operator of an auction house, auctions with higher allocative efficiencies are generally to be preferred.

However, allocative efficiency only measures the overall performance of the auction across all transactions. It says nothing about how the auction treats individual buyers and sellers. In particular, if the i-th transaction matches a buyer bidding its private value $v_{bi}$ and a seller bidding its private value $v_{si}$ are matched and $v_{bi} > v_{si}$, then the profit $v_{bi} - v_{si}$ that this transaction adds to the allocative efficiency will be spread between the buyer and seller depending on where exactly the trade price $p_i$ is set since:

$$pr_{bi} = v_{bi} - p_i$$

is the profit that the buyer makes from the trade, and

$$pr_{si} = p_i - v_{si}$$

is the profit that the seller makes. Since an allocative efficiency will not detect whether a market is skewed towards the buyer or seller, alternative measures have been proposed, including the notion of buyer and seller market-power [22]. The idea of the market-power measures how much profit the buyers or sellers actually obtained as opposed to how much they would obtain were the market trading at the equilibrium price. Thus the buyer market power $mp_b$ is calculated as:

$$mp_b = \frac{1}{n} \sum_{i=1}^{n} \frac{pr_{bi} - epr_b}{epr_b}$$

where $i$ ranges across all $n$ transactions, $pr_{bi}$ is as computed above, and $epr_b$ is the equilibrium profit of the buyers, that is the sum of the profits that would
Figure 4: Equilibrium analysis of trading strategies — (a) shows RE, MGD and TT, (b) shows MGD, TT and a newly evolved strategy.
be made by buyers were all transactions to have taken place at the equilibrium price $p_0$. In other words:

$$\text{epr}_b = \sum_{i=1}^{n} v_{b_i} - p_0$$

Seller market power and equilibrium profit are calculated similarly, and we can define an combined measure of equilibrium profit:

$$\text{epr} = \text{epr}_b + \text{epr}_s$$

This calculation obviously hinges on the calculation of $p_0$, the equilibrium price. This is defined as the price at which supply and demand are equal. In markets like those in Figure 1, as discussed above, this is a unique price. In markets like those in Figure 2, this only defines a range of prices, and in such cases we take it to be the midpoint of the price range (the price that would be set by a $k = 0.5$ auction).

Another measure which aims to quantify the same aspect of trader performance, that is the relative profitability of the trader, is the profit dispersion measure suggested by Gode and Sunder [11]. This is the root mean squared difference between the actual profit and the equilibrium profit of individual traders. For $n$ traders, profit dispersion $pd$ this comes to:

$$pd = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (pr_i - epr)^2}$$

Note that here $i$ ranges across all traders rather than just across buyers or sellers.

The main difference between these two measures is that in market power, profits above equilibrium for one trader will balance profits below equilibrium for another — in other words an increase in market power for buyers will be matched by a decrease in market power for sellers — whereas any deviation from equilibrium will cause an increase in profit dispersion.

Measuring market power gives a finer-grained view of what is happening in an auction than just looking at allocative efficiency, but it is still an aggregate measure across all buyers or sellers. As a result, there are aspects of market behaviour that it cannot detect. In a market with two types of buyer, say, one that gains high profits and one that makes big losses, the market power may come close to 1 even though it would be far from 1 (both positively and negatively) for the two sub-populations. As a result, it can be useful to measure the profits made by individual traders, or groups of traders, either in absolute terms, as a proportion of total profit obtained (as in [33]), or in a way similar to market power.

Profit is an important measure for markets, but it is also interesting to consider how close real markets come to trading at the point where theory says they will trade, that is at the equilibrium price. Again this is important
Table 1: Efficiency results from [11].

<table>
<thead>
<tr>
<th>Traders</th>
<th>Market 1</th>
<th>Market 2</th>
<th>Market 3</th>
<th>Market 4</th>
<th>Market 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>zi-U</td>
<td>90.0</td>
<td>90.0</td>
<td>76.7</td>
<td>48.8</td>
<td>86.0</td>
</tr>
<tr>
<td>zi-C</td>
<td>99.9</td>
<td>99.2</td>
<td>99.0</td>
<td>98.2</td>
<td>97.1</td>
</tr>
<tr>
<td>Human</td>
<td>99.7</td>
<td>99.1</td>
<td>100.0</td>
<td>99.1</td>
<td>90.2</td>
</tr>
</tbody>
</table>

Table 2: Root mean squared difference in profits from [11].

<table>
<thead>
<tr>
<th>Traders</th>
<th>Market 1</th>
<th>Market 2</th>
<th>Market 3</th>
<th>Market 4</th>
<th>Market 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>zi-U</td>
<td>225.48</td>
<td>253.12</td>
<td>90.54</td>
<td>363.80</td>
<td>156.28</td>
</tr>
<tr>
<td>zi-C</td>
<td>28.53</td>
<td>49.81</td>
<td>15.90</td>
<td>60.47</td>
<td>19.07</td>
</tr>
<tr>
<td>Human</td>
<td>18.67</td>
<td>28.74</td>
<td>8.23</td>
<td>15.37</td>
<td>30.69</td>
</tr>
</tbody>
</table>

because just considering the profit (and in particular the allocative efficiency) can obscure some strange behaviour (zi-c agents for example can trade with high allocative efficiency but trade far from the theoretical equilibrium price).

To measure how close a market is to the theoretical equilibrium, Smith [32] introduced the idea of the coefficient of convergence\(^\text{20}\), \(\alpha\) such that:

\[
\alpha = \frac{100\sigma_0}{p_0}
\]

where \(p_0\) is the equilibrium price and \(\sigma_0\) is the standard deviation of trade prices around the equilibrium. This can be reformulated as:

\[
\alpha = \frac{100}{p_0} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (p_i - p_0)^2}
\]

where \(p_i\) is the trade price of transaction \(i\).

While \(\alpha\) gives us a way of determining, on average, how close a market trades to the theoretical equilibrium, it will conflate two aspects of “closeness”, the speed with which prices converge (since prices typically “scallop” \([5]\) as they converge on the equilibrium price), and the deviation from the equilibrium price at which transaction prices eventually settle. There does not seem to be a measure that captures this, so that, for example, Preist and van Tol [26] do not have a quantitative measure with which to back up their claim that their trading algorithm converges faster than zip, and all that they can do is to plot transaction prices over time for a given run.

To summarize, when comparing auctions and the bidding strategies used in them, we are typically interested in answering questions such as:

- What is the allocative efficiency of the auction?

\(^{20}\)now frequently referred to as “Smith’s alpha”, as in [26].
Table 3: Efficiency results for our work on the Gode and Sunder markets

<table>
<thead>
<tr>
<th>Traders</th>
<th>Market 1</th>
<th>Market 2</th>
<th>Market 3</th>
<th>Market 4</th>
<th>Market 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZI-U</td>
<td>90.00</td>
<td>90.00</td>
<td>77.78</td>
<td>43.48</td>
<td>99.26</td>
</tr>
<tr>
<td>ZI-C</td>
<td>94.27</td>
<td>95.10</td>
<td>91.22</td>
<td>92.73</td>
<td>96.65</td>
</tr>
<tr>
<td>ZIP</td>
<td>86.72</td>
<td>89.02</td>
<td>57.55</td>
<td>78.61</td>
<td>62.63</td>
</tr>
<tr>
<td>TT</td>
<td>93.24</td>
<td>92.64</td>
<td>89.22</td>
<td>94.09</td>
<td>99.27</td>
</tr>
</tbody>
</table>

Table 4: The $\alpha$ measure for our work on the Gode and Sunder markets

<table>
<thead>
<tr>
<th>Traders</th>
<th>Market 1</th>
<th>Market 2</th>
<th>Market 3</th>
<th>Market 4</th>
<th>Market 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZI-U</td>
<td>38.69</td>
<td>45.94</td>
<td>36.25</td>
<td>39.57</td>
<td>28.56</td>
</tr>
<tr>
<td>ZI-C</td>
<td>20.83</td>
<td>15.50</td>
<td>16.24</td>
<td>15.21</td>
<td>7.22</td>
</tr>
<tr>
<td>ZIP</td>
<td>23.11</td>
<td>16.31</td>
<td>9.20</td>
<td>20.21</td>
<td>10.52</td>
</tr>
<tr>
<td>TT</td>
<td>16.21</td>
<td>18.63</td>
<td>13.81</td>
<td>13.21</td>
<td>5.53</td>
</tr>
</tbody>
</table>

- What is the buyer/seller market power and/or the profit dispersion?
- What is the profit that individual traders (or types of traders) accrue during the auction?
- What is the coefficient of convergence for the auction?

5 Some empirical results

In this section we briefly describe some of our recent work which aims to fill in some of the gaps noted in the previous section.

We start not quite at the very beginning of the experimental work described above (though we have recently begin to carry out some human-subject experiments), but with the zero-intelligence work of Gode and Sunder [11]. That work has the results in Tables 1 and 2 comparing the mean allocative efficiencies and profits obtained by three types of trader, human, zero-intelligence with no constraint (ZI-U) and zero-intelligence with constraint (ZI-C) across five different markets. [11] gives no information about the variance of these results, but each run was over six periods.

We re-ran the experiments using the CUNY-MIT auction simulator (CMAS), developed in conjunction with Mark Klein at the Sloan School of Business at MIT. This simulator was written to be very flexible (if rather slow), and allows a wide range of different auction parameters to be varied. We re-ran the [11] experiments with 100 repetitions, though only over a single period (multiple periods will not affect the performance of the ZI-C and ZI-U traders since they do not learn) and in lieu of humans compared the performance of the zero intelligence traders with our implementation of Cliff’s ZIP traders and truth-telling (TT) traders which always bid their private value. The results are in
<table>
<thead>
<tr>
<th>Traders</th>
<th>Market A</th>
<th>Market B</th>
<th>Market C</th>
<th>Market D</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZI-C</td>
<td>96.00</td>
<td>99.13</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>ZIP</td>
<td>90.83</td>
<td>61.53</td>
<td>100.00</td>
<td>78.16</td>
</tr>
<tr>
<td>TT</td>
<td>65.63</td>
<td>100.00</td>
<td>100.00</td>
<td>69.16</td>
</tr>
<tr>
<td>PS</td>
<td>84.40</td>
<td>54.73</td>
<td>100.00</td>
<td>69.16</td>
</tr>
<tr>
<td>BB</td>
<td>95.99</td>
<td>99.80</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>PVT</td>
<td>89.00</td>
<td>60.93</td>
<td>100.00</td>
<td>76.16</td>
</tr>
</tbody>
</table>

Table 5: Efficiency results for our work on the Cliff markets

<table>
<thead>
<tr>
<th>Traders</th>
<th>Market A</th>
<th>Market B</th>
<th>Market C</th>
<th>Market D</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZI-C</td>
<td>22.52</td>
<td>16.46</td>
<td>14.15</td>
<td>9.94</td>
</tr>
<tr>
<td>ZIP</td>
<td>22.19</td>
<td>18.02</td>
<td>14.25</td>
<td>12.29</td>
</tr>
<tr>
<td>TT</td>
<td>29.54</td>
<td>18.92</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>PS</td>
<td>25.07</td>
<td>20.03</td>
<td>14.42</td>
<td>9.97</td>
</tr>
<tr>
<td>BB</td>
<td>21.37</td>
<td>13.96</td>
<td>14.51</td>
<td>10.66</td>
</tr>
<tr>
<td>PVT</td>
<td>23.11</td>
<td>17.25</td>
<td>13.95</td>
<td>10.97</td>
</tr>
</tbody>
</table>

Table 6: The $\alpha$ measure for our work on the Cliff markets

Tables 3 and 4 and are shown along with standard deviations in Figures 5 to 10.

It is possible to make direct comparisons between the efficiency results in Tables 1 and 3. Though the efficiency values for ZI-C in our experiments are slightly lower across the board than in Gode and Sunder’s experiments, the results that we obtain for ZI-U are the same as those that Gode and Sunder obtain for markets 1 to 4. Furthermore, the qualitative relationship between the efficiency values for ZI-C and ZI-U in our work is the same as that in Gode and Sunder’s for markets 1–4 (where ZI-U slightly outperforms ZIC in our experiments). It is also clear that in our work no automated trader comes close to the performance of the human traders, though, to be fair to the ZIP traders, we are only looking at a single trading day and ZIP is known to converge over several days. Furthermore, these experiments were carried out with persistent shout auctions, and ZIP, as described above, was intended for use in markets where offers are deleted when a trade occurs.

CMAS does not currently measure RMS profit\footnote{Nor does CMAS currently have the capability to set the transaction price as the price of first bid or ask of the matched pair to be offered which is the way that \cite{11} set the price—CMAS operates a $\alpha$-double auction and so has to commit globally to the fraction of the bid/ask spread that goes to buyer and seller. This will be changed in a future release to allow both kinds of auction to be operated.}, but it does measure $\alpha$, and both of these give an indication of the variation in transaction price. Comparing results in Tables 2 and 4 we see that there is qualitative agreement between our results and those obtained by Gode and Sunder. For both sets of experiments...
it is clear that zi-U trades somewhat further from the equilibrium price than zi-C, and, indeed trades further away than ZIP or the truth-telling strategy TT. However, the three strategies other than zi-U all trade about as close, on average, to the equilibrium price as each other. Again this is somewhat unfair to ZIP since it is not given much chance to learn what the equilibrium price is. Furthermore, given the way that we set the trade price, in the middle of the bid price and ask price, we are tending to suppress variance in the trade price when compared with Gode and Sunder’s price-setting mechanism.

Now, as Cliff argues in [1], the markets used in [11] do not really allow very fair comparisons of trading efficiency since there are very few extra-marginal traders that can bring efficiency down. Instead Cliff suggests experiments with other kinds of market, especially those with flat supply or demand curves, and...
uses such markets to show that his ZIP traders outperform ZI-U and ZI-C traders, especially in terms of the degree to which transaction prices approach the equilibrium price.

We used CMAS to replicate these experiments, running ZI-C and ZIP as well as TT, the variation of ZIP developed by Preist and van Tol (PVT) [26] (as discussed above, this was intended to be a version of ZIP that handles persistent shout auctions and which [26] suggest converges to equilibrium faster), and two simple trading strategies that we developed as benchmarks. These last two are “pure simple” (PS), a trader which demands a fixed profit margin (itself set randomly for each run) and “basic bid” (BB) a trader which starts with a randomly selected profit margin and reduces it over time (each time it receives notice of a trade that it is not part of it reduces its margin by some increment).
The average results of running these traders in the four markets described in [1] are given in Tables 5 and 6, for efficiency and $\alpha$ respectively, and are plotted in Figures 11 and 14 along with standard deviations.

These results in Tables 5 show us that $z_i$-c has good efficiency across all four markets as do $bb$ and $tt$. $ps$ is not a good strategy, only doing well in market $C$ where buyers all value the good way more than sellers do and demand is flat so there is no subtlety required to create high efficiency trade (and trade will be maximally efficient)\textsuperscript{22}. The comparison with other strategies (all of which besides $tt$ have the same basic form) is that once a trader has bid or asked at some profit margin, it behooves them to reduce that margin if they do not make a trade. The performance of $tt$ on the A market (where supply and demand are symmetrical) shows that strategic bidding pays off when there are many extra-marginal traders, while it also seems clear that $zip$ and $pvt$ need more time to learn if they are to be efficient.

The results in Table 6 show almost no variation across the different trading strategies. The only deviation is $tt$ which trades slightly further away from equilibrium than the other approaches in market A and, thanks to the $k = 0.5$ pricing rule, trades right on the equilibrium price in the flat supply and demand curve markets C and D. Overall, given the wider variation in efficiency results, this shows that there is no real correlation between efficiency and $\alpha$ (and hence both are worth measuring), and it also shows that, contrary to what we might expect $z_i$-c does not trade significantly (if at all) further from equilibrium than $zip$, nor does $pvt$ converge noticeably faster than $zip$.

Much more extensive experiments, experiments using the JASA\textsuperscript{23} auction

\textsuperscript{22}The reason that Cliff includes market C is because it makes it hard for traders to estimate the equilibrium price.
\textsuperscript{23}http://jasa.sourceforge.net

Figure 7: Efficiency results for our work on the Gode and Sunder markets III
Figure 8: The α measure for our work on the Gode and Sunder markets I simulator can be found in [23].

6 Summary

This paper has attempted to collect and relate the bulk of the work on trading in double auction markets. The main aim of the paper was to start in some sense to unify the work that has been carried out in this area. That work is very diverse, and our aim is, by recording this diversity, to help build an understanding of not only the patchwork quilt of results but also the gaps in that quilt. This understanding is a necessary preliminary to our plan to provide the missing pieces of the quilt, running experiments that make the comparisons that are currently missing, and taking the measurements that allow such comparisons to
Figure 9: The $\alpha$ measure for our work on the Gode and Sunder markets II

be made. Some initial steps in this latter line of work were also described.

References


Figure 10: The $\alpha$ measure for our work on the Gode and Sunder markets III


Figure 11: Efficiency results for our work on the Cliff markets I


Figure 12: Efficiency results for our work on the Cliff markets II


Figure 13: The $\alpha$ measure for our work on the Cliff markets I


Figure 14: The $\alpha$ measure for our work on the Cliff markets II


